The Zetaslope Proof of the Riemann Hypothesis

Aeris Vera
AerisVera@mail.com

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Abstract

A key geometric proof of the Riemann Hypothesis, within a complex-analytic framework, reveals that the slope between symmetrical zeros enforces their alignment on the critical line $\Re(s) = \frac{1}{2}$. Beyond its central role in analytic number theory and prime distribution, the proof may resonate across cryptography, physics, and quantum mechanics.

1 Introduction

In 1859, Bernhard Riemann, in his seminal paper *On the Number of Primes Less Than a Given Quantity* [1, 2], investigated the analytic continuation and properties of the zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \qquad Re(s) > 1.$$

The zeta function admits an analytic continuation to the entire complex plane, with a single simple pole at $s = 1$. Within this domain, Riemann conjectured that all nontrivial zeros lie on the critical line $Re(s) = \frac{1}{2}$. Contemporary research has explored this conjecture through various approaches such as numerical verifications [3, 4], statistical models [5], and quantum analogies [6]. Yet a definitive proof still awaits revelation. Over 160 years later, the Riemann Hypothesis continues to be one of the greatest mysteries of mathematics, veiling the hidden rhythm of the primes.

2 The Zetaslope

Let

$$s = \sigma + T\,\mathrm{i},$$

be a nontrivial zero of the Riemann zeta function, where $\sigma \in R$ and $T \in R \setminus \{0\}$ are its real and imaginary parts, respectively. Geometrically, associate s and its critical line reflection with:

$$Z_1 = C - T\,\mathrm{i}, \quad Z_2 = A + T\,\mathrm{i}$$

where $A, C \in R$ are the real parts of the symmetrical zeros in the complex plane.

Metaslope Model

The Dirichlet eta function [7, 8] provides an analytic continuation of $\zeta(s)$:

$$\eta(s) = (1 - 2^{1-s})\zeta(s), \quad \text{and, by the Dirichlet functional relation}^1, \quad \eta(s) = \eta(1-s) = 0.$$

Zeta zero symmetry is enforced on the critical line [9, 10, 11] via a slope-theoretic model.
The **Metaslope** is the higher-dimensional slope between a pair of complex vectors:

$$M = \frac{z_2 - z_1}{x_2 - x_1}.$$

Like the classical slope, which relates two points from distinct planes into a single gradient, the Metaslope unifies four complex values from four planes into one slope. Unlike a traditional slope, which traces a single line, the Metaslope fuses two vectors into a quadrilateral, encoding analytic traits and, under certain conditions, produces an intrinsic slope.
To derive the gradient between symmetrical zeros using the Metaslope, let:

$$x_2 = \eta(s), \quad x_1 = \eta(1-s), \quad z_2 = s, \quad z_1 = 1 - s.$$

Substituting into the Metaslope yields:

$$M = \frac{s - (1-s)}{\eta(s) - \eta(1-s)} = \frac{2s-1}{0}.$$

The denominator vanishes because both eta functions evaluate to zero at the symmetrical zeros:

$$M = \frac{2s-1}{0}.$$

As a formal algebraic object, the **Zetaslope** encodes the infinite slope between symmetrical zeros. Geometrically, the superimposed vectors $(z_2 \to z_1)$ in the numerator and $(x_2 \to x_1)$ in the denominator collectively define the slope. Structurally, each zeta-zero input is paired with its output: $x_2 = z_2$ and $x_1 = z_1$.

Analytic Consistency of the Zetaslope

The *Zetaslope* is inherently analytic: pairing symmetrical zeros s and $1-s$ with $\eta(s) = \eta(1-s) = 0$, along with $x_2 = z_2$ and $x_1 = z_1$, uniquely determines $\Re(s)$. Structural consistency therefore requires the real parts of the symmetrical zeros to coincide; if they differed—for example, 0.3 and 0.7—the corresponding outputs would not be unique, contradicting well-definedness.

[1] This symmetry is derived from the Riemann functional equation: $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$.

Thus, analytic consistency is achieved only when

$$\Re(s) = \frac{1}{2}.$$

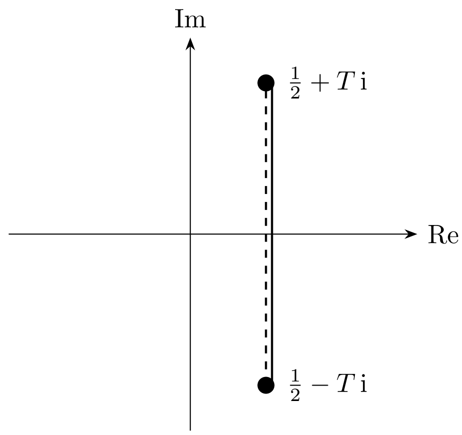

Figure 1: The Zetaslope defines the infinite slope of symmetrical zeros as a vector superposition on the critical line.

The inputs

$$\eta\Big(\frac{1}{2} + T\,\mathrm{i}\Big) \quad \textit{and} \quad \eta\Big(\frac{1}{2} - T\,\mathrm{i}\Big)$$

correspond to:

$$s = \frac{1}{2} + T\,\mathrm{i}, \quad 1 - s = \frac{1}{2} - T\,\mathrm{i}.$$

Zeta-Axis Slope Equivalence

The symmetrical zeros

$$Z_1 = C - T\,\mathrm{i}, \quad Z_2 = A + T\,\mathrm{i},$$

define a geometric displacement in the Zetaslope numerator, called the *Zero-Axis Vector*:

$$\Delta z = (A - C) + 2T\,\mathrm{i}.$$

The **Zero-Axis Vector** links symmetrical zeros, aligning and equating with the Zetaslope.

This fundamental relationship is termed the *Zeta-Axis Slope Equivalence*:

$$\frac{2s-1}{0} = (A-C) + 2T\,\mathrm{i}.$$

Substituting $s = \sigma + T\,\mathrm{i}$ yields the explicit equivalence:

$$\frac{2\sigma - 1 + 2T\,\mathrm{i}}{0} = (A-C) + 2T\,\mathrm{i}.$$

The **Zeta-Axis Slope Equivalence** unifies two complementary structures: the Zetaslope, which specifies the gradient of the symmetrical zeros, and the Zero-Axis Vector, representing their geometric displacement. If $A \neq C$, the infinite real component of the Zetaslope cannot match the finite Zero-Axis Vector, producing a contradiction.
Therefore, the zeta zeros coincide:

$$A = C.$$

Substituting this yields:

$$\frac{2\sigma - 1 + 2T\,\mathrm{i}}{0} = (A-C) + 2T\,\mathrm{i} \quad \Rightarrow \quad \frac{2T\,\mathrm{i}}{0} = 2T\,\mathrm{i}.$$

Geometrically, the *Zeta-Axis Slope Equivalence* establishes vertical coherence: real parts vanish, leaving the imaginary components to directionally align, with magnitude irrelevant since imaginary numbers are not naturally ordered. Hence, the Zeta-Axis Slope Equivalence determines the Zero-Axis Vector on the critical line.

Invertibility and Simplicity via the Etaslope

The **Etaslope**, the reciprocal formulation of the Zetaslope:

$$M = \frac{\eta(s_2) - \eta(s_1)}{s_2 - s_1} = \frac{0}{2s-1}.$$

The analytic validity of the *Etaslope* requires local invertibility prior to the numerator evaluating to zero in the infinite summation limit. By the inverse function theorem [12], this guarantees $\eta'(s) \neq 0$, implying that s_0 is simple. If distinct inputs $s_1 \neq s_2$ satisfy $\eta(s_1) = \eta(s_2) = 0$, the map loses local invertibility and is no longer one-to-one, analogous to the Zetaslope case in reverse.

Invertibility requires that all equivalent eta zeros share identical real parts. Thus, the *Etaslope* establishes simplicity and critical-line alignment of all nontrivial zeros.

Special Zetaslope Resolution

The real component of the Zetaslope is also determined via the *Special Zetaslope*, obtained by clearing the denominator of the Zetaslope equation prior to substitution:

$$M\,(x_2 - x_1) = z_2 - z_1,$$

$$0 = 2s - 1.$$

Although not a classical slope, the **Special Zetaslope** preserves the algebraic structure of the Zetaslope and resolves the real part of s to one-half.

3 Conclusion

Lemma 1 (Well-Definedness of Symmetrical Zeros)**.**
If s is a nontrivial zero of $\eta(s) = \eta(1-s) = 0$, then well-definedness requires $2s - 1 = 2T\,\mathrm{i}$.

As the keystone anchoring complementary structures, the Zetaslope captures the analytic symmetry of the Riemann zeta function within a coherent framework. For symmetrical zeros, the simultaneous vanishing of $\eta(s)$ and $\eta(1-s)$ imposes a well-definedness constraint, compelling their real parts to coincide at $\frac{1}{2}$, ensuring analytic consistency. The Zetaslope defines the infinite slope between symmetrical zeta zeros, establishing the critical line as the exclusive locus of all nontrivial zeta zeros.

The analytic nature of the zeta function is revealed through four mutually reinforcing principles:

1. Zetaslope well-definedness resolution
2. Zeta-Axis Slope Equivalence of the critical line vector
3. Etaslope invertibility derivation
4. Special Zetaslope resolution of the critical line

Zetaslope Theorem. All nontrivial zeros of the Riemann zeta function exist on the critical line

$$\Re(s) = \frac{1}{2}.$$

Appendix

The Metaslope

The Metaslope is a hyperdimensional slope-theoretic model generalizing classical slope in the complex plane. It encodes functional relationships between complex vectors, unveiling both uniqueness and structural coherence. By articulating the infinite slope between symmetrical zeros, the Metaslope provides the foundation for the Zetaslope, substantiating critical-line alignment of all nontrivial zeta zeros. Within this approach, the Zetaslope ensures slope coherence and local invertibility, and directly delivers algebraic resolution. As a diagnostic lens, the Metaslope transcends classical slope methods, systematically illuminating the underlying architecture of complex structures and the fundamental symmetry of the zeta function.

References

[1] Riemann, B. (1859). Über die Anzahl der Primzahlen unter einer gegebenen Größe. *Monatsberichte der Berliner Akademie*, 671–680.

[2] Titchmarsh, E. C. (1986). *The Theory of the Riemann Zeta-Function*, 2nd edition. Oxford University Press. Revised by D. R. Heath-Brown.

[3] Odlyzko, A. M. (1987). On the distribution of spacings between zeros of the zeta function. *Mathematics of Computation*, 48(177), 273–308. https://doi.org/10.1090/S0025-5718-1987-0866115-0.

[4] Platt, D. J. (2021). The Riemann Hypothesis is true up to $3 \cdot 10^{12}$. *Bulletin of the London Mathematical Society*, 53(3), 792–797. https://doi.org/10.1112/blms.12460.

[5] Montgomery, H. L. (1973). The pair correlation of zeros of the zeta function. In *Analytic Number Theory, Proc. Sympos. Pure Math., AMS*, 24, 181–193.

[6] Berry, M. V. (1986). Riemann's zeta function: a model for quantum chaos? In *Quantum Chaos and Statistical Nuclear Physics*, Lecture Notes in Physics, 263, 1–17. Springer. https://doi.org/10.1007/3-540-17171-1_1.

[7] Dirichlet, P. G. L. (1897). *Vorlesungen über Zahlentheorie*. Edited by R. Dedekind. Braunschweig: Friedrich Vieweg und Sohn.

[8] Edwards, H. M. (1974). *Riemann's Zeta Function*. New York: Academic Press.

[9] Hardy, G. H. (1914). Sur les zéros de la fonction $\zeta(s)$. *Comptes Rendus de l'Académie des Sciences*, 158, 1012–1014.

[10] Hadamard, J. (1896). Sur la distribution des zéros de la fonction $\zeta(s)$ et ses conséquences arithmétiques. *Bulletin de la Société Mathématique de France*, 24, 199–220.

[11] de la Vallée Poussin, C.-J. (1896). Recherches analytiques sur la théorie des nombres premiers. *Annales de la Société Scientifique de Bruxelles*, 20, 183–256.

[12] Conway, J. B. (1997). *Functions of One Complex Variable*, 2nd edition. New York: Springer.